Optimal replication of von Neuman measurements

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Suitable mathematical representation of objects emerging in quantum mechanics is crucial for solving most of the optimization problems. Introduction of Process Positive Operator Valued Measures (PPOVM) and Quantum Combs allowed to solve several problems in which the most general (thought) experiments involving N uses of a tested quantum channel (completely positive trace preserving maps) have to be optimized. In this contribution, we apply quantum combs to optimize quantum circuits achieving transformations of measurements. More precisely, such a circuit has to work as one big POVM after N measurements are inserted into the open slots of the circuit. The aim of the circuit is to create M replicas of the inserted measurements, which are assumed to be unknown POVMs of the Von Neuman type (i.e. non-degenerate projective measurements). We show that for arbitrary figure of merit the presence of measurements in the circuit allow us to restrict the optimization to a subclass of quantum combs, which are called diagonal. Using diagonal quantum combs we solve the following tasks: N→ 1, 1→2 Learning of a qudit POVM (N=1,2,3) and 1→2 Cloning of a qudit POVM. The goal of N→M Learning of a POVM is to use the unknown measurement N times, store what was learned about it in a quantum memory and later retrieve M uses of the original measurement on a state that is not available in the learning phase. In contrast in N→M Cloning of a POVM the state to be measured is available from the very beginning, but we have to mimic M>N uses of the unknown measurement by using it just N times. We compare the performance of the optimal 1→2 Learning with 1→2 Cloning of a POVM. Similarly, to the analogous tasks for unitary channels the performance of cloning is much better than that of learning. We discovered that the uses of the unknown measurements in the optimal circuit cannot be parallelized for 3→1 Learning of a qudit POVM. Thus, N→1 Learning of a qudit POVM represents a task, where the optimal strategy is necessarily sequential. This feature of non-parallelizability is present also in Grover algorithm, where the calls to the oracle cannot be parallelized as was shown by Zalka. Indirectly, our findings can help to understand how to search for optimal quantum circuits i.e. optimal quantum algorithms with oracle callings, which cannot be parallelized.