The quantum and classical embedding problems

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The embedding problem is a long-standing open problem in probability theory, dating at least as far back as 1937. The problem is to characterise of the stochastic maps that can be generated by a continuous-time Markov process. A very similar question can be asked in the quantum setting, where the problem becomes one of characterising the completely-positive maps that can be generated by a master equation.

From the literature, these may appear to be very abstract mathematical problems. Far from it! The embedding problem (and its quantum generalisation) is closely related to a very practical task in experimental physics. Imagine that you have gathered a large amount of measurement data for some physical system, whose behaviour you would like to understand. The embedding problem is essentially the problem of using that experimental data to reconstruct the dynamical equations of the system.

In recent work, we [1,2] finally layed both the classical and quantum embedding problems to rest, by proving that they are NP-hard. I will explain the embedding problems and their relation to physics, outline their recent resolution, and discuss their implications.

[1] T. S. Cubitt, J. Eisert, M. M. Wolf, arXiv:0908.2128[math-ph]
[2] T. S. Cubitt, J. Eisert, M. M. Wolf, arXiv:1005.0005[quant-ph]